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无穷多海洋表面波相互作用的能量守恒和共振条件*

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摘要: 依照能量守恒定律和业已证明的海洋表面波之波-波共振条件,通过将 Hamilton 能量泛函展开至一个 7 阶对称的积分幂级数,给出了一个典型的"3-4-5-6-7 波相互作用系统的共振条件组".进而归纳、推论出一个一般的"无穷多波相互作用系统的共振条件组",据此可显著地改观目前的基本海洋波湍流理论格局.

关 键 词: 共振条件; 无穷多海洋表面波; 能量守恒; 对称性; 波湍流
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引 言

现代水波理论肇始于 1960 年提出的海洋表面重力波之 4-波共振条件[1],随后于 1962 年就触发了最为基本、普遍的海洋波湍流——以统计系综平均描述的 Hasselmann 方程[2],再至 1968 年就产生了以普适的 Hamilton 结构刻画 4-波共振之确定性的 Zakharov 方程[3].可以证明,由 Zakharov 方程出发,即能推导出 Hasselmann 方程[4].

显见,贯穿上述水波理论发展史的脉络为"共振条件",并以"Hamilton 结构"和"对称性"[5-10]为不可或缺.

如果从一部理论物理学的"基本相互作用"发展近现代史[11-13]来看,将凸显出若干为数不多的"基本物理本性",而这必将包含"能量守恒定律"和"对称性决定相互作用[12]".那么,试问:能否将这两大通用物理法则与普适的 Hamilton 结构相结合而应用于多个乃至"无穷多个海洋表面波共振条件"的求解之道?

果如是,那将是自然而然地,并将突破时至今日为获得海洋波动的经典"3-波[14]、4-波[1]、5-波[15]"之"共振条件"而惯用的普遍旧有模式.

本文试为之.

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1 能量守恒

弱非线性海洋表面波的 Hamilton 复正则方程[3]为

$$\mathrm{i}\frac{\partial a(\boldsymbol{k},t)}{\partial t}=\frac{\delta H(a(\boldsymbol{k},t),a^*(\boldsymbol{k},t))}{\delta a^*(\boldsymbol{k},t)}, \tag{1}$$

其中, H 即为刻画表面波系统能量的 Hamilton 泛函, a^* 乃复正则函数 a 之共轭, $\boldsymbol{k}$ 是波数矢量, δ 代表泛函导数. 为比较完整地保证能量守恒,且遵循"对称性决定相互作用"[12]的根本原理,可对 H 实施满足"自然对称性"的积分幂级数之超过目前最高 5 阶[16-17]的 7 阶展开:

$$\begin{aligned}
H=&\int\omega_0a_0a_0^*\,\mathrm{d}\boldsymbol{k}_0+\\
&\int\left\{\left[U^{(1)}_{0,1,2}a_0^*a_1a_2\delta_{0-1-2}+\frac{1}{3}U^{(3)}_{0,1,2}a_0^*a_1^*a_2^*\delta_{0+1+2}\right]+\text{c.c.}\right\}\mathrm{d}\boldsymbol{k}_{012}+\\
&\int\left\{\frac{1}{2}V^{(2)}_{0,1,2,3}a_0^*a_1^*a_2a_3\delta_{0+1-2-3}+\right.\\
&\left.\left[V^{(1)}_{0,1,2,3}a_0^*a_1a_2a_3\delta_{0-1-2-3}+\frac{1}{4}V^{(4)}_{0,1,2,3}a_0^*a_1^*a_2^*a_3^*\delta_{0+1+2+3}\right]+\text{c.c.}\right\}\mathrm{d}\boldsymbol{k}_{0123}+\\
&\int\left\{\left[W^{(1)}_{0,1,2,3,4}a_0^*a_1a_2a_3a_4\delta_{0-1-2-3-4}+\frac{1}{2}W^{(2)}_{0,1,2,3,4}a_0^*a_1^*a_2a_3a_4\delta_{0+1-2-3-4}+\right.\right.\\
&\left.\left.\frac{1}{5}W^{(5)}_{0,1,2,3,4}a_0^*a_1^*a_2^*a_3^*a_4^*\delta_{0+1+2+3+4}\right]+\text{c.c.}\right\}\mathrm{d}\boldsymbol{k}_{01234}+\\
&\int\left\{\frac{1}{3}X^{(3)}_{0,1,2,3,4,5}a_0^*a_1^*a_2^*a_3a_4a_5\delta_{0+1+2-3-4-5}+\right.\\
&\left[X^{(1)}_{0,1,2,3,4,5}a_0^*a_1a_2a_3a_4a_5\delta_{0-1-2-3-4-5}+\frac{1}{2}X^{(2)}_{0,1,2,3,4,5}a_0^*a_1^*a_2a_3a_4a_5\delta_{0+1-2-3-4-5}+\right.\\
&\left.\left.\frac{1}{6}X^{(6)}_{0,1,2,3,4,5}a_0^*a_1^*a_2^*a_3^*a_4^*a_5^*\delta_{0+1+2+3+4+5}\right]+\text{c.c.}\right\}\mathrm{d}\boldsymbol{k}_{012345}+\\
&\int\left\{\left[Y^{(1)}_{0,1,2,3,4,5,6}a_0^*a_1a_2a_3a_4a_5a_6\delta_{0-1-2-3-4-5-6}+\right.\right.\\
&\frac{1}{2}Y^{(2)}_{0,1,2,3,4,5,6}a_0^*a_1^*a_2a_3a_4a_5a_6\delta_{0+1-2-3-4-5-6}+\\
&\frac{1}{3}Y^{(3)}_{0,1,2,3,4,5,6}a_0^*a_1^*a_2^*a_3a_4a_5a_6\delta_{0+1+2-3-4-5-6}+\\
&\left.\left.\frac{1}{7}Y^{(7)}_{0,1,2,3,4,5,6}a_0^*a_1^*a_2^*a_3^*a_4^*a_5^*a_6^*\delta_{0+1+2+3+4+5+6}\right]+\text{c.c.}\right\}\mathrm{d}\boldsymbol{k}_{0123456},
\end{aligned} \tag{2}$$

其中, ω_0 为描述有限水深表面张力-重力波色散关系的角频率,c.c.代表前项的复共轭,而各个因变函数的下标 j 代指自变量 $\boldsymbol{k}_j$, 且下标 0 专指 $\boldsymbol{k}$, 例如, $a_j=a(\boldsymbol{k}_j,t)$, $U^{(n)}_{0,1,2}=U^{(n)}(\boldsymbol{k},\boldsymbol{k}_1,\boldsymbol{k}_2)$, δ_j 表示 Dirac 函数 $\delta(\boldsymbol{k}_j)$.另外, $\mathrm{d}\boldsymbol{k}_{012}=\mathrm{d}\boldsymbol{k}_0\mathrm{d}\boldsymbol{k}_1\mathrm{d}\boldsymbol{k}_2$,各个积分号表示在整个 $\boldsymbol{k}$- 平面从"$-\infty$"到"$+\infty$"的相应多重积分.

依据式(1)和(2),可得 a 的演化方程:

$$\mathrm{i}\frac{\partial a_0}{\partial t}=\omega_0a_0+\int\left[U^{(1)}_{0,1,2}a_1a_2\delta_{0-1-2}+2U^{(1)}_{2,0,1}a_1^*a_2\delta_{0+1-2}+U^{(3)}_{0,1,2}a_1^*a_2^*\delta_{0+1+2}\right]\mathrm{d}\boldsymbol{k}_{12}+$$

$$
\begin{aligned}
&\int\left[V^{(1)}_{0,1,2,3}a_1a_2a_3\delta_{0-1-2-3}+V^{(2)}_{0,1,2,3}a_1^*a_2a_3\delta_{0+1-2-3}+3V^{(1)}_{3,0,1,2}a_1^*a_2^*a_3\delta_{0+1+2-3}+\right.\\
&\left.V^{(4)}_{0,1,2,3}a_1^*a_2^*a_3^*\delta_{0+1+2+3}\right]\mathrm{d}\boldsymbol{k}_{123}+\\
&\int\left[W^{(1)}_{0,1,2,3,4}a_1a_2a_3a_4\delta_{0-1-2-3-4}+W^{(2)}_{0,1,2,3,4}a_1^*a_2a_3a_4\delta_{0+1-2-3-4}+\right.\\
&\frac{3}{2}W^{(2)}_{3,4,0,1,2}a_1^*a_2^*a_3a_4\delta_{0+1+2-3-4}+4W^{(1)}_{4,0,1,2,3}a_1^*a_2^*a_3^*a_4\delta_{0+1+2+3-4}+\\
&\left.W^{(5)}_{0,1,2,3,4}a_1^*a_2^*a_3^*a_4^*\delta_{0+1+2+3+4}\right]\mathrm{d}\boldsymbol{k}_{1234}+\\
&\int\left[X^{(3)}_{0,1,2,3,4,5}a_1^*a_2^*a_3a_4a_5\delta_{0+1+2-3-4-5}+X^{(1)}_{0,1,2,3,4,5}a_1a_2a_3a_4a_5\delta_{0-1-2-3-4-5}+\right.\\
&5X^{(1)}_{5,0,1,2,3,4}a_1^*a_2^*a_3^*a_4^*a_5\delta_{0+1+2+3+4-5}+X^{(2)}_{0,1,2,3,4,5}a_1^*a_2a_3a_4a_5\delta_{0+1-2-3-4-5}+\\
&2X^{(2)}_{4,5,0,1,2,3}a_1^*a_2^*a_3^*a_4a_5\delta_{0+1+2+3-4-5}+\\
&\left.X^{(6)}_{0,1,2,3,4,5}a_1^*a_2^*a_3^*a_4^*a_5^*\delta_{0+1+2+3+4+5}\right]\mathrm{d}\boldsymbol{k}_{12345}+\\
&\int\left[Y^{(1)}_{0,1,2,3,4,5,6}a_1a_2a_3a_4a_5a_6\delta_{0-1-2-3-4-5-6}+\right.\\
&6Y^{(1)}_{6,0,1,2,3,4,5}a_1^*a_2^*a_3^*a_4^*a_5^*a_6\delta_{0+1+2+3+4+5-6}+\\
&Y^{(2)}_{0,1,2,3,4,5,6}a_1^*a_2a_3a_4a_5a_6\delta_{0+1-2-3-4-5-6}+\\
&\frac{5}{2}Y^{(2)}_{5,6,0,1,2,3,4}a_1^*a_2^*a_3^*a_4^*a_5a_6\delta_{0+1+2+3+4-5-6}+\\
&Y^{(3)}_{0,1,2,3,4,5,6}a_1^*a_2^*a_3a_4a_5a_6\delta_{0+1+2-3-4-5-6}+\\
&\frac{4}{3}Y^{(3)}_{4,5,6,0,1,2,3}a_1^*a_2^*a_3^*a_4a_5a_6\delta_{0+1+2+3-4-5-6}+\\
&\left.Y^{(7)}_{0,1,2,3,4,5,6}a_1^*a_2^*a_3^*a_4^*a_5^*a_6^*\delta_{0+1+2+3+4+5+6}\right]\mathrm{d}\boldsymbol{k}_{123456}.
\end{aligned}
\tag{3}
$$

由式(2)和(3),可以证明能量守恒:

$$
\frac{\partial H}{\partial t}=0. \tag{4}
$$

2 共振条件

相对于多种物理守恒定律(例如,电荷、重子守恒等),能量守恒定律显得更为超拔、抽象,实则就是一种数学原理[11].能量可有多种物理表达形式(例如声、光、电、热能等),那么,其守恒定律是否也存在多种数学形式呢?并且,由于“能量守恒是一种局域过程”[11],那么,在数学的整体表达式上是否也能显现出某种“局域过程”?这需要观察、验证、推断.现在,可将式(4)局域化:分解、展开而另表述为

$$
\frac{\partial H}{\partial t}=\left[\mathrm{i}\int\omega_0^2a_0a_0^*\,\mathrm{d}\boldsymbol{k}_0+A+B\right]-\left[\mathrm{i}\int\omega_0^2a_0a_0^*\,\mathrm{d}\boldsymbol{k}_0+A+B\right]=A-A=0, \tag{5}
$$

其中, B 包含的项数远大于 A,每一项又含有两个相异的 δ 之积(须做平均而消去一个 δ),而 A 中的各项仅含有一个 δ,且较之 B 还含有若干个角频率的组合,即为

$$
\begin{aligned}
\mathrm{i}A=\int&\left[U^{(1)}_{0,1,2}(a_0^*a_1a_2-\mathrm{c.c.})(\omega_0-\omega_1-\omega_2)\delta_{0-1-2}+\right.\\
&\left.\frac{1}{3}U^{(3)}_{0,1,2}(a_0^*a_1^*a_2^*-\mathrm{c.c.})(\omega_0+\omega_1+\omega_2)\delta_{0+1+2}\right]\mathrm{d}\boldsymbol{k}_{012}+
\end{aligned}
$$

$$
\begin{aligned}
&\int\Big[\frac{1}{2}V^{(2)}_{0,1,2,3}a_0^*a_1^*a_2a_3(\omega_0+\omega_1-\omega_2-\omega_3)\delta_{0+1-2-3}+\\
&V^{(1)}_{0,1,2,3}(a_0^*a_1a_2a_3-\text{c.c.})(\omega_0-\omega_1-\omega_2-\omega_3)\delta_{0-1-2-3}+\\
&\frac{1}{4}V^{(4)}_{0,1,2,3}(a_0^*a_1^*a_2^*a_3^*-\text{c.c.})(\omega_0+\omega_1+\omega_2+\omega_3)\delta_{0+1+2+3}\Big]\mathrm{d}\boldsymbol{k}_{0123}+\\
&\int\Big[\frac{1}{2}W^{(2)}_{0,1,2,3,4}(a_0^*a_1^*a_2a_3a_4-\text{c.c.})(\omega_0+\omega_1-\omega_2-\omega_3-\omega_4)\delta_{0+1-2-3-4}+\\
&W^{(1)}_{0,1,2,3,4}(a_0^*a_1a_2a_3a_4-\text{c.c.})(\omega_0-\omega_1-\omega_2-\omega_3-\omega_4)\delta_{0-1-2-3-4}+\\
&\frac{1}{5}W^{(5)}_{0,1,2,3,4}(a_0^*a_1^*a_2^*a_3^*a_4^*-\text{c.c.})(\omega_0+\omega_1+\omega_2+\omega_3+\omega_4)\delta_{0+1+2+3+4}\Big]\mathrm{d}\boldsymbol{k}_{01234}+\\
&\int\Big[\frac{1}{3}X^{(3)}_{0,1,2,3,4,5}a_0^*a_1^*a_2^*a_3a_4a_5(\omega_0+\omega_1+\omega_2-\omega_3-\omega_4-\omega_5)\delta_{0+1+2-3-4-5}+\\
&X^{(1)}_{0,1,2,3,4,5}(a_0^*a_1a_2a_3a_4a_5-\text{c.c.})(\omega_0-\omega_1-\omega_2-\omega_3-\omega_4-\omega_5)\delta_{0-1-2-3-4-5}+\\
&\frac{1}{2}X^{(2)}_{0,1,2,3,4,5}(a_0^*a_1^*a_2a_3a_4a_5-\text{c.c.})(\omega_0+\omega_1-\omega_2-\omega_3-\omega_4-\omega_5)\delta_{0+1-2-3-4-5}+\\
&\frac{1}{6}X^{(6)}_{0,1,2,3,4,5}(a_0^*a_1^*a_2^*a_3^*a_4^*a_5^*-\text{c.c.})(\omega_0+\omega_1+\\
&\omega_2+\omega_3+\omega_4+\omega_5)\delta_{0+1+2+3+4+5}\Big]\mathrm{d}\boldsymbol{k}_{012345}+\\
&\int\Big[\frac{1}{3}Y^{(3)}_{0,1,2,3,4,5,6}(a_0^*a_1^*a_2^*a_3a_4a_5a_6-\text{c.c.})(\omega_0+\omega_1+\\
&\omega_2-\omega_3-\omega_4-\omega_5-\omega_6)\delta_{0+1+2-3-4-5-6}+\\
&Y^{(1)}_{0,1,2,3,4,5,6}(a_0^*a_1a_2a_3a_4a_5a_6-\text{c.c.})(\omega_0-\omega_1-\\
&\omega_2-\omega_3-\omega_4-\omega_5-\omega_6)\delta_{0-1-2-3-4-5-6}+\\
&\frac{1}{2}Y^{(2)}_{0,1,2,3,4,5,6}(a_0^*a_1^*a_2a_3a_4a_5a_6-\text{c.c.})(\omega_0+\omega_1-\\
&\omega_2-\omega_3-\omega_4-\omega_5-\omega_6)\delta_{0+1-2-3-4-5-6}+\\
&\frac{1}{7}Y^{(7)}_{0,1,2,3,4,5,6}(a_0^*a_1^*a_2^*a_3^*a_4^*a_5^*a_6^*-\text{c.c.})(\omega_0+\omega_1+\\
&\omega_2+\omega_3+\omega_4+\omega_5+\omega_6)\delta_{0+1+2+3+4+5+6}\Big]\mathrm{d}\boldsymbol{k}_{0123456}.
\end{aligned}
\tag{6}
$$

一般而言,数学共振条件仅包含角频率条件,而物理共振条件则是“角频率和波数矢量共存于一体”.因此,这里的物理共振条件可由 A 担当,并且在“波数矢量的组合” 上 ,显然 B 的种类多于且包含 A .如果令:$A=0$,则 A 之式(6)右端的 16 项各自均为 0.据此,可得出一个完整的“3-4-5-6-7 波相互作用系统的共振条件组”:

$$\omega_0=\omega_1+\omega_2,\ \boldsymbol{k}_0=\boldsymbol{k}_1+\boldsymbol{k}_2, \tag{7a}$$

$$\omega_0+\omega_1+\omega_2=0,\ \boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3=\boldsymbol{0}; \tag{7b}$$

$$\omega_0+\omega_1=\omega_2+\omega_3,\ \boldsymbol{k}_0+\boldsymbol{k}_1=\boldsymbol{k}_2+\boldsymbol{k}_3, \tag{8a}$$

$$\omega_0=\omega_1+\omega_2+\omega_3,\ \boldsymbol{k}_0=\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3, \tag{8b}$$

$$\omega_0+\omega_1+\omega_2+\omega_3=0,\ \boldsymbol{k}_0+\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3=\boldsymbol{0}; \tag{8c}$$

$$\omega_0+\omega_1=\omega_2+\omega_3+\omega_4,\ \boldsymbol{k}_0+\boldsymbol{k}_1=\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4, \tag{9a}$$

$$\omega_0=\omega_1+\omega_2+\omega_3+\omega_4,\ \boldsymbol{k}_0=\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4, \tag{9b}$$

$$\omega_0+\omega_1+\omega_2+\omega_3+\omega_4=0,\ \boldsymbol{k}_0+\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4=\boldsymbol{0}; \tag{9c}$$

$$\omega_0+\omega_1+\omega_2=\omega_3+\omega_4+\omega_5,\ \boldsymbol{k}_0+\boldsymbol{k}_1+\boldsymbol{k}_2=\boldsymbol{k}_3+\boldsymbol{k}_4+\boldsymbol{k}_5, \tag{10a}$$

$$\omega_0=\omega_1+\omega_2+\omega_3+\omega_4+\omega_5,\ \boldsymbol{k}_0=\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4+\boldsymbol{k}_5, \tag{10b}$$

$$\omega_0+\omega_1=\omega_2+\omega_3+\omega_4+\omega_5,\ \boldsymbol{k}_0+\boldsymbol{k}_1=\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4+\boldsymbol{k}_5, \tag{10c}$$

$$\omega_0+\omega_1+\omega_2+\omega_3+\omega_4+\omega_5=0,\ \boldsymbol{k}_0+\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4+\boldsymbol{k}_5=\boldsymbol{0}; \tag{10d}$$

$$\omega_0+\omega_1+\omega_2=\omega_3+\omega_4+\omega_5+\omega_6,\ \boldsymbol{k}_0+\boldsymbol{k}_1+\boldsymbol{k}_2=\boldsymbol{k}_3+\boldsymbol{k}_4+\boldsymbol{k}_5+\boldsymbol{k}_6, \tag{11a}$$

$$\omega_0=\omega_1+\omega_2+\omega_3+\omega_4+\omega_5+\omega_6,\ \boldsymbol{k}_0=\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4+\boldsymbol{k}_5+\boldsymbol{k}_6, \tag{11b}$$

$$\omega_0+\omega_1=\omega_2+\omega_3+\omega_4+\omega_5+\omega_6,\ \boldsymbol{k}_0+\boldsymbol{k}_1=\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4+\boldsymbol{k}_5+\boldsymbol{k}_6, \tag{11c}$$

$$\begin{cases}\omega_0+\omega_1+\omega_2+\omega_3+\omega_4+\omega_5+\omega_6=0,\\ \boldsymbol{k}_0+\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4+\boldsymbol{k}_5+\boldsymbol{k}_6=\boldsymbol{0}.\end{cases} \tag{11d}$$

在上述共振条件中,式(7b),(8c),(9c),(10d),(11d)为 5 个“平凡共振条件”,意味着负能量波的传播介质[6],可给予排除;式(7a),(8a),(9a)为目前业已发现的共振条件,依次代表“表面张力-重力波的 3-波共振[14]”、“表面重力波的 4-波[1]、5-波[15]共振”.由此,可以推测出上述“非凡共振条件组”的“归属类型”:

(Ⅰ) 表面张力-重力波

式(7a),(8b),(9b),(10b),(11b);

(Ⅱ) 表面重力波

式(8a),(9a),(10a),(10c),(11a),(11c).

如果欲将上述“非凡共振条件组”扩展至 n- 波状态,则可归纳推断出一般的“无穷多波相互作用系统的非凡共振条件组”:

(Ⅰ) 表面张力-重力波

$$\omega_1=\omega_2+\omega_3+\cdots+\omega_n,\ \boldsymbol{k}_1=\boldsymbol{k}_2+\boldsymbol{k}_3+\cdots+\boldsymbol{k}_n,\qquad n\geqslant 3; \tag{12}$$

(Ⅱ) 表面重力波

$$\begin{cases}\omega_1+\omega_2+\cdots+\omega_m=\omega_{m+1}+\omega_{m+2}+\cdots+\omega_n,\\ \boldsymbol{k}_1+\boldsymbol{k}_2+\cdots+\boldsymbol{k}_m=\boldsymbol{k}_{m+1}+\boldsymbol{k}_{m+2}+\cdots+\boldsymbol{k}_n,\end{cases}\qquad 2\leqslant m\leqslant n-2. \tag{13}$$

可以推断,如果将上述“波能量守恒定律”代之于相应的“波动量守恒定律”或“波作用量守恒定律”[6],势必亦可推演出“非凡共振条件组”(12)和(13).

3 结　　论

本文着眼并联合如下基本、普适的物理“概念、定律、方法和法则”:“共振、能量守恒定律、Hamilton 描述、对称性决定相互作用”,一举、完整、有效获得了“无穷多波相互作用系统的共振条件组”.这不但为将经典 4-波共振的 Zakharov 方程[3]和 Hasselmann 方程[2]推广至无穷多波共振而提供了一个先决理论基础,进而将显著地促进广泛的海洋波湍流[2,16-19]乃至普适波湍流[6,8,20-21]的研究和发展,而且或许提供了一个“求证多种多样波动(例如光波、声波和交通波等)的多波共振条件”的新方法论——这又将触及到丰富多彩的物理守恒量:动量、角动量和绝热不变量(adiabatic invariants)[22]等.

如果欲将上述理论远景拉回到近处,便会发现眼下这样一个迫切的实际要求:“将目前海洋波湍流之最多的 5-波共振[2,16-19]推广至 6-波,即要得到海洋表面波之 6-波共振的 Zakharov 方程及其 Hasselmann 方程,以及该 Hasselmann 方程的某一能量谱特解”.该特解类同于与湍流

中著名的“Kolmogorov 谱”或“Kolmogorov-Obukhov 谱”[6,23-25]相似的 Kolmogorov-Zakharov 谱[26]. 正是后者，一举将“波湍流”严格置身于“湍流”的大框架体系中去了[6,20-21].

显然，这将是极其艰难却又十分难得的海洋波湍流一大进展，甚或波湍流.

本作者试为之.

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Energy Conservation and Resonance Conditions for Interactions of an Infinite Number of Ocean Surface Waves

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Abstract: Based on the energy conservation law and the existing wave-wave resonance conditions for ocean surface waves, a typical group of resonance conditions for the 3-4-5-6-7 wave interactions was put forward through expansion of the Hamiltonian energy functional into a 7-order symmetrical integro-power series, therefore a general group of resonance conditions for an infinite number of wave interactions was induced and deduced. The work may make a great improvement in the present structure of the fundamental wave turbulence theory.

Key words: resonance condition; an infinite number of ocean surface waves; energy conservation; symmetry; wave turbulence

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